K_A = adsorption equilibrium constant for alcohol
K_H = adsorption equilibrium constant for hydrogen
K_K = adsorption equilibrium constant for ketone
L = total molal adsorption sites per unit mass
M = parameter, $(1+K_Ap_A)/\sqrt{D}$
p_A = partial pressure of alcohol, atm.
p_K = partial pressure of ketone, atm.
p_H = partial pressure of hydrogen, atm.
r = rate of reaction, lb.-moles/(hr.)(lb. of catalyst)
r_{A_0} = initial rate of reaction from a feed containing only alcohol
r_{AH_0} = initial rate of reaction from a feed containing alcohol and hydrogen
r_{AK_0} = initial rate of reaction from a feed containing alcohol and ketone
x_b,x_c,x_d = independent variables for Equation (1)
y = dependent variable for Equation (1)
π = total pressure, atm.

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A Study of Sieve-Tray Efficiencies

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The performance of sieve trays in the rectification of the methanol-water system without entrainment or leakage from the perforations was studied in an 8-in.–diameter five-tray column. The trays had a 2-in. weir height and 4-in. length of liquid path. Three tray geometries were studied: 1/4-in. diameter holes on 3/4-in. triangular spacing, 1/8-in. holes on 3/8-in. triangular spacing, and 3/16-in. on 7/16-in. triangular spacing. The superficial vapor velocity was varied from 2.2 ft./sec. to the limit of stable operation, which for this apparatus was 4.4 ft./sec. The ratio L_M/V_M within the column was varied from 1 to 0.5. The Murphree plate efficiency varied greatly from 105% at low concentration to 82% at high concentrations of methanol. Variations of 10 or 12 efficiency % were noted owing to changing velocities and tray geometries. Measurements of concentration gradients, foam heights, and gas pressure drops are also reported. This paper proposes a method of calculating the point efficiency and the number of individual-phase mass transfer units independent of the actual concentration gradient on the tray.

The method is applied to the methanol-water data, and calculated point efficiencies range from 50 to 65%. The value of $1/N_L$ for the methanol-water system is found to be small. The values of N_G and the effect of the velocity on N_G are believed to be the first in the literature for a tray in distillation operation. The effect of velocity is shown to be in agreement with the theory proposed by Gerster and co-workers. It is shown that $k_G' a_G$ decreases for increasing free area and increasing hole size. Finally variation in L_M/V_M is shown to have little effect on E_{MV}.

Fractionation in tray type of distillation columns is perhaps the most widely used means of separating the components of a liquid mixture. The calculation of the number of trays required for such a separation can be made readily (*31*) even for the most complicated separations. However only in recent years has much effort been directed toward analyzing the efficiency of operation of such trays in view of modern mass transfer theory (*1, 5, 6, 9, 14, 16, 17, 23, 27*). Any such analysis is complicated by the large number of variables involved in the properties of the gas and liquid phases, the conditions of operation, and the geometry of the system.

This paper reports efficiency data obtained by operating an 8-in. sieve-tray column at total reflux on the system methanol-water and discusses the data in the light of recent theory. A special study is made of the number of gas-phase transfer units.

The definitions of the transfer unit terms, appearing in the notation, will be assumed known to the reader. The usual assumptions made in using them are that the driving forces are concentration difference for liquids and partial-pressure difference for vapors and that equilibrium exists at phase interfaces. In addition in this paper it will be assumed that the vapor composition below a tray is uniform, that there is no back mixing of the vapor in the vertical direction, and that there are no unusual end effects at either bubble formation or break.

THE ADDITION OF RESISTANCES

The basic equation for obtaining the individual gas-phase value depends on the well-known (*32*) equation describing the additivity of resistances:

$$\frac{1}{N_{OG}} = \frac{1}{N_G} + \frac{m_G V_M}{L_M}\frac{1}{N_L} \qquad (1)$$

Normally in using this equation one assumes N_G and N_L to be constant and plots $1/N_{OG}$ against $(m_GV_M)/L_M$ to obtain N_G from an intercept on such a plot. Using total-reflux sieve-tray plate-efficiency data, one must modify this direct approach in two ways.

First with total reflux data V_M/L_M has a value of unity, and $(m_GV_M/L_M$

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TABLE 1. TABULATION OF TRAY GEOMETRIES

Tray type	Position in column	Hole diameter, in.	Triangular spacing, center to center, in.	No. of moles	Free area, sq. ft.	% free area based on Bubbling area	% free area based on Column area
I	1	1/8	3/8	216	0.0185	8.56	5.23
II	2	3/16	7/16	145	0.0278	12.87	8.02
III	3	1/4	3/4	49	0.0167	7.73	4.82
II	4	3/16	7/16	149	0.0286	13.21	8.23
I	5	1/8	3/8	218	0.0186	8.61	5.36
IIA	1	3/16	3/8	151	0.0290	13.41	8.34
IIIA	1	1/4	3/4	73	0.0249	11.51	17.37

Velocities used: Series *A* 5.8 moles/hr. *B* 6.8 moles/hr. *C* 9.4 moles/hr. *D* 11.3 moles/hr.

can be changed only by changing the concentration on the trays, which will affect the values of N_G and N_L. It is necessary, therefore, to introduce into Equation (1) correction factors similar to those used by Oliver and Watson (*33*), which allow for the effect of these variables:

$$\frac{1}{N_{OG}B} = \frac{1}{N_{GS}} + \frac{m_G V_M}{L_M} \frac{A}{B} \frac{1}{N_{LS}} \quad (2)$$

where, from the definitions of N_L and N_G

$$A = \frac{N_{LS}}{N_L} = \frac{k_{LS}}{k_L} \frac{a_{LS}}{a_L} \frac{\epsilon_S}{\epsilon} \frac{q}{q_S} \quad (3)$$

$$B = \frac{N_{GS}}{N_G} = \frac{k_{GS}'}{k_G} \frac{a_{GS}}{a_G} \frac{\beta_S}{\beta} \frac{U}{U_S} \quad (4)$$

An equation similar to (2) can be written for the liquid phase:

$$\frac{1}{N_{OL}A} = \frac{1}{N_{LS}} + \frac{1}{\dfrac{m_L V_M}{L_M} \dfrac{A}{B} N_{GS}} \quad (5)$$

The second factor to be considered, and one which still prevents the direct application of the revised Equation (2), is that the type of mixing on a sieve tray is not known. Thus N_{OG} applying at a point cannot be obtained directly from the observed tray efficiencies. It is proposed that a pseudo number of transfer units be defined as the N_{OG} value calculated from the composition change across the tray on the assumption that each of the following cases is applicable:

(*a*) The liquid may be completely mixed; then

$$N_{OG} = -\ln(1 - E_{MV}) \quad (6)$$

(*b*) As considered by Lewis (*24*), the liquid may be completely mixed in the vertical direction but not mixed in the direction of liquid flow; then

$$E_{OV} = \frac{1}{\dfrac{m_T V_M}{L_M}} \ln \frac{m_T V_M}{L_M} E_{MV} + 1 \quad (7)$$

where E_{OV} is related to the value of N_{OG} by an equation similar to (6).

(c) As described by Kamei and Takamatsu (*23*) there may be no mixing in the direction of either liquid or vapor flow. These authors presented graphically the relationship between E_{MV}, $(m_T V_M)/L_M$, Z_V, Z_H, and N_{OG}.

To obtain N_{OL} for Equation (5) two special cases are applicable:

(a) If it is assumed that the liquid moves across the plate in plug flow and that the equilibrium concentration x^* remains constant on the tray, then

$$N_{OL} = -\ln(1 - E_{ML}) \quad (8)$$

(b) If it is assumed that the liquid on the tray is completely mixed and the equilibrium concentration remains constant, then

$$N_{OL} = \frac{E_{ML}}{1 - E_{ML}} \quad (9)$$

If there is no mixing on the tray in either phase, the Kamei and Takamatsu relationship between E_{ML}, N_{OL}, $(m_T V_M)/L_M$, Z_H, and Z_V discussed above will apply.

In the case of extrapolation to $1/N_G$ all the special cases should have the same intercept since they all assume that there is no mixing of the vapor in the direction of vapor flow. This assumption is reasonable since, in general, vapor bubbles will rise through a liquid with little tendency for back mixing.

However for the case of extrapolation to $1/N_{LS}$ no such assumption is applicable, for it is exactly the variation among types of mixing that necessitates the theoretical approach proposed in this paper. Thus the intercept corresponding to Equation (8) will be different from that of Equation (9), and the Kamei and Takamatsu intercept would be expected to be the same as for Equation (8). The two intercepts should give the limits of N_{LS}. It may be added here that for most practical cases $1/N_{LS}$ will be small and will have little effect on the calculated point efficiency except for very large values of m_G.

TABLE 2. CONCENTRATION GRADIENTS

(Concentration in mole per cent methanol)

Run	Location of sampler	Reflux sample Before	Reflux sample After	Sampler location 1	Sampler location 2	Sampler location 3	Liquid leaving plate Before	Liquid leaving plate After	Reflux reading	Temperature, °C Top	Temperature, °C return	Production reading	Top plate L_M/V_M	Vapor flow top moles/hr.
31	Vapor above tray	90.5	89.4	86.4	85.6	82.6	75.2	73.2	45.5	68.7	30.5	—	—	12.45
	Foam liquid			—	78.4	74.6								
	Liquid on tray floor			77.4	73.4	69.3								
	Vapor below tray			79.4	76.4	73.7								
34	Vapor above tray			—	78.6	74.1								
	Foam liquid			—	65.1	55.7								
	Liquid on tray floor	79.9	77.4	51.6	49.1	48.1		60.7	40.9*	73.6	23.7	—	—	11.80*
	Liquid on tray floor	77.4	74.8	40.4	35.9	34.9		56.1						
	Vapor below tray			—	60.3	53.0								
36	Vapor above tray	52.9	51.1	59.1	54.4	50.6	17.7	14.4	30.0*	82.5	23.0	—	—	10.13*
	Foam liquid			35.0	25.4	21.2								
	Liquid on tray floor			16.3	11.4	7.0								
	Vapor below tray			27.1	23.0	19.4								

* Average values.

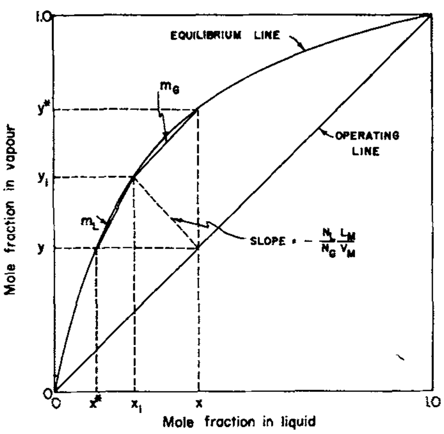

Fig. 1. Representation of mass transfer driving forces on x-y diagram.

It will be noted that N_{OG}' is calculated for a concentration change taking place across a whole tray, whereas m_G, A, and B refer to mass transfer taking place at particular liquid and vapor compositions.

Actually these quantities are calculated at the average liquid and vapor compositions across the tray, the concentration changes being small, and thus only a very small error is introduced.

The method as developed to this point could be used with data obtained from a number of systems each of which had a different constant value of m_G. However it seems more realistic at the present time to employ a single system having a variation in m_G. In this latter case the effect of the physical-property corrections is small. For boiling systems the equilibrium line will be curved, and it is necessary to calculate m_G, m_L, and m_T for the curved equilibrium line.

It is easily shown that the following define m_G and m_L exactly:

$$m_G = \frac{y^*-y_i}{x-x_i};\ m_L = \frac{y_i-y}{x_i-x^*} \quad (10\,a,b)$$

where x_i and y_i are equilibrium values on the line of slope $-N_L/N_G$ passing through x, y. It is therefore necessary to determine x_i and y_i, which requires a trial-and-error solution. To do this the ratio N_{LS}/N_{GS} is first assumed and N_L/N_G calculated for other compositions. N_{LS} and N_{GS} are then determined by extrapolation, and the original assumption is checked.

For use in the Lewis equation, for a curved equilibrium line the following is assumed:

$$m_T = \frac{y^*_{x\,\text{on}} - y^*_{x_\text{off}}}{x_\text{on}-x_\text{off}} \quad (11)$$

Figure 1 has been included to illustrate the various slopes m_G, m_L, and m_T referred to in this paper.

For the special case of Kamei and Takamatsu it was assumed that m_T' could be approximated by Equation (11); this gave reasonable results.

EXPERIMENTAL APPARATUS

The experimental apparatus and the trays employed are illustrated in Figures 2 and 3. Table 1 lists the various tray geometries employed. A standard distillation circuit was used, the overhead product being recycled to the reboiler. A complete description is given in reference *19*. The top tray was located in a glass cylinder for visual observation of foam height and leakage.

Samples were taken from the downcomers as illustrated in Figure 4, by method similar to that of Garner *et al.* (*13*). The method of sampling and analysis must be as precise as possible mainly because of the apparent magnification of errors caused by taking differences in Equations (5) and (6).

The top tray was used for a study of concentration gradients. Samples were withdrawn from the floor of the tray, from the foam ½ in. above the floor, and from the vapor above and below the tray by means of fixed sampling probes in the floor (Figure 3) and a movable probe for the liquid and vapor.

Foam heights were measured visually by using the top tray. All tray types were installed in the top position during the course of the experimental work to obtain foam heights and concentration gradients.

Pressure drops across the trays were also measured. After correction for the dry-tray pressure drop these were used to obtain foam densities.

Thermometers were installed in the vapor above all trays. In addition the temperature above tray 4 was recorded. Only after all temperatures and flows had been steady for at least ½ hr. were the samplers closed off and samples taken; a steady temperature was defined as a temperature constant within ±0.25°C. Usually the apparatus required 2 to 5 hr. to reach steady conditions; during this time the column was operated manually.

DISCUSSION OF RESULTS

The column was operated at four vapor velocities, denoted on the graphs as series A, B, C, and D, ranging from 5.85 to 11.34 moles/hr. Data were taken on the three tray geometries. There was no entrainment at the highest velocity that could be detected by injecting dye on one of the intermediate trays and sampling the liquid from the tray above. Also there was no visible leakage from any of the tray types at the lowest velocity.

CONCENTRATION GRADIENTS

Table 2 shows some typical concentration gradients. In interpreting these measured concentrations, one should note that when vapor samples and samples are obtained from the foam, a slight amount of the undesirable phase may be sampled. This is more critical for the vapor than the liquid, since a small amount of entrained liquid could have a great effect on the measured concentration.

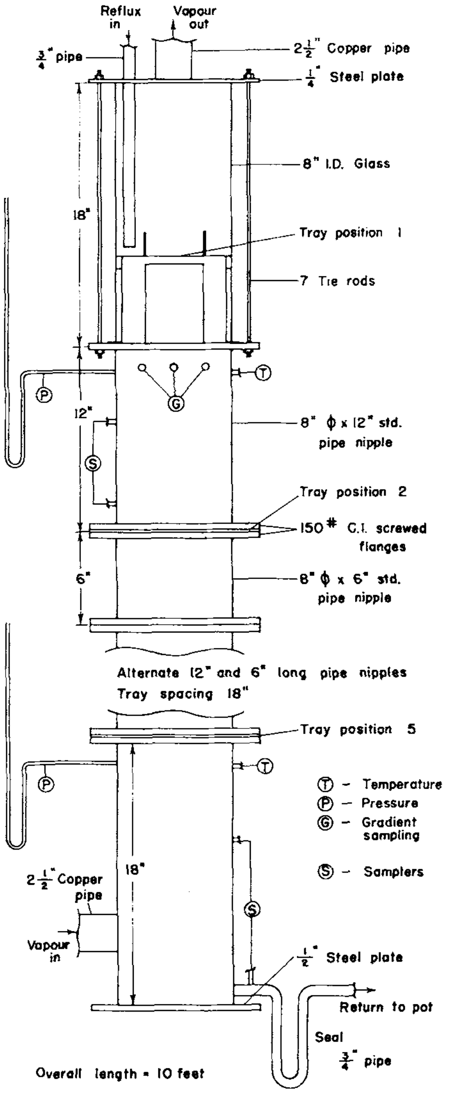

Fig. 2. Detail drawing of column.

Vapor concentration gradients were evident for the runs where the total change in concentration across the tray was large. However the effect of these vapor gradients on tray performance must be small, since the same tray performs identically in positions 1 and 5 in the column; position 5 has of course a homogeneous vapor below it.

Examination of the liquid concentration gradients shows that in some runs the concentration of the liquid on the tray floor is lower than that of the liquid leaving the tray. Further, the concentration in the foam above the tray floor is always higher than on the floor. These data therefore suggest a vertical concentration gradient in the liquid worthy of further study.

It was felt that the trays in this column were too short to warrant any attempt to use the concentration gradients to study mixing theory.

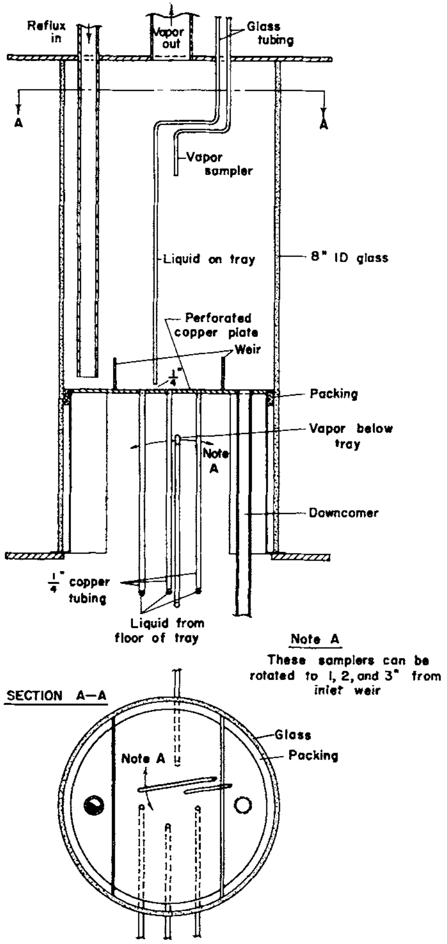

Fig. 3. Concentration-gradient sampling.

FOAM PROPERTIES

The volumetric gas holdup and clear liquid holdup on the tray are required for the calculation of contact time. Foam heights and pressure drops were measured at the four velocities and for the three tray geometries shown in Table 1. A typical plot of the data is shown in Figure 5. The scatter in the data is large at the low weir height used, and the two low-velocity curves *A* and *B* and the two high-velocity curves *C* and *D* cannot be separated. The foam pressure drop ΔP in centimeters of water is shown on the plots together with $\Delta P_f'$ in terms of the average liquid density on the tray.

The foam volume ratio is the ratio of clear liquid height to foam height. Although its accurate calculation is difficult because of the errors in measurement, a value of ϕ' is calculated by using Z_v and $\Delta P_f'$. Here ϕ' is proportional to ϕ provided that the difference between ΔP_f and the clear liquid head is constant (*3, 22*). Other effects such as the influence of interfacial tension have been neglected in the calculation of ϕ' in this paper.

The purpose of the foam study presented here was to determine the variation of foam density and foam height with gas rate, composition, and free area for the methanol-water system. The data are seen to support the assumption that the gas and liquid holdup do not vary significantly with composition for the system studied. Also ϕ is constant with varying gas velocity, but the foam height increases with increasing gas velocity. The effect of hole size is negligible. However for the tray with increased free area the foam density decreased, since the foam height was almost the same for all trays; β increased and ϵ decreased for this tray.

In a separate study Price (*30*) has reported increasing value of ϕ' as the weir height was increased.

It is recognized that these conclusions are debatable because of the scatter of the data. However they are also supported by other data in the literature (*4, 27*) (taken from nonboiling trays) with the notable exception of the data of Gerster and Foss (*17*). These latter authors reported decreasing ϕ values with increasing gas velocity, but it is not clear whether or not the effects of free-area change have been eliminated.

While the ϕ values calculated in this study are not sufficiently accurate to resolve controversies regarding the factors influencing them, it is believed that final values of β/U shown on Figure 11, which depend on the ϕ values, are accurate to ± 0.01.

TRAY-PERFORMANCE DATA AT TOTAL REFLUX

Because the calculation of efficiencies from experimental data magnifies errors, the authors would like to discuss their treatment of the data in detail. It should be noted that at total reflux the concentration of liquid leaving a tray is the same as the vapor entering it. It is postulated that the concentration change across a tray will be a unique, continuous function of the average concentration of the liquid on the plate. Therefore the experimental data were plotted as $Y_{off} - Y_{on}$ against the arithmetic average concentration Y_{avg}.

Here Y_{off} and Y_{on} are the average mole fractions of more volatile in the vapor leaving or entering a tray and are obtained from the liquid samples, since the column is operated at total reflux.

An average curved line was drawn through the data on such plots, as illustrated in Figure 6. Efficiencies were then calculated from those smoothed curves for all further calculations.

These curves were drawn for all plates for all vapor velocities. Complete data are given in reference *19*. The method described above was then applied for each geometry and velocity with a standard state of 60 mole % methanol used. This reference concentration was chosen because $1/N_G$ was

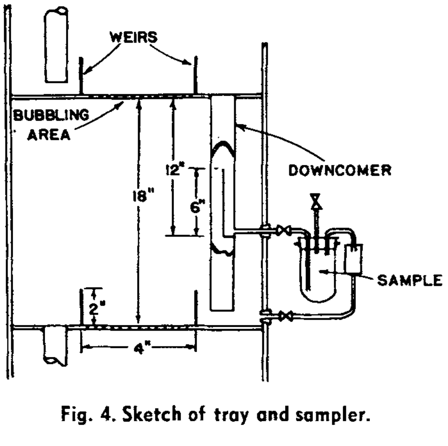

Fig. 4. Sketch of tray and sampler.

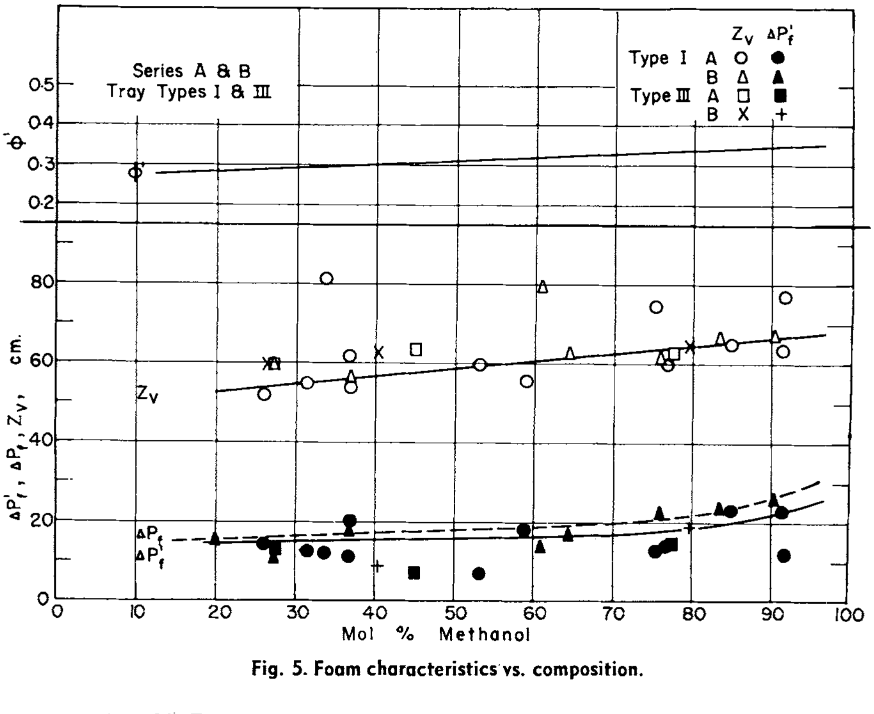

Fig. 5. Foam characteristics vs. composition.

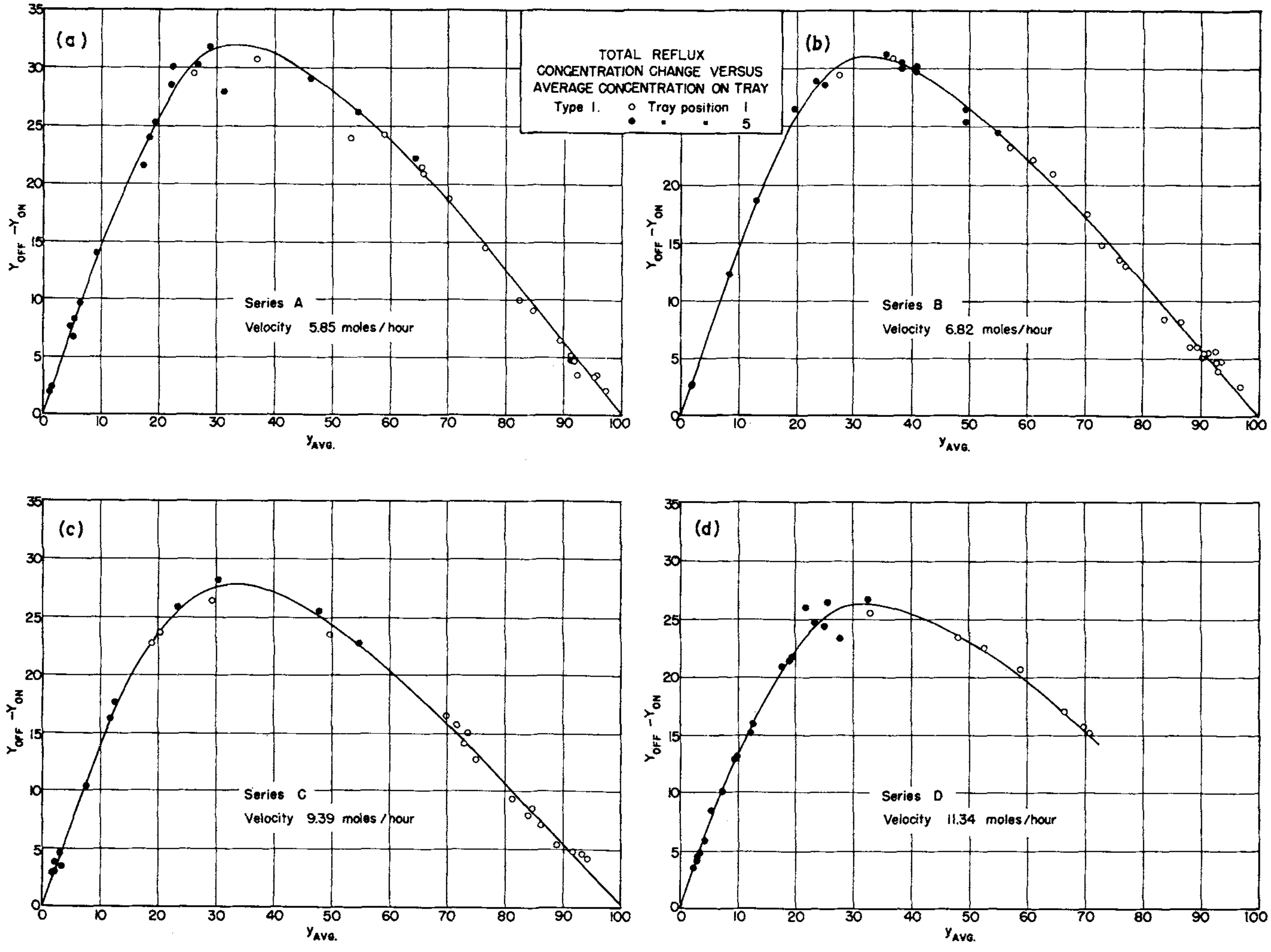

Fig. 6. Tray performance.

practically as large and $1/N_L$ was larger than at any other concentration.

The method of calculation uses the following steps:

(a) Using the smoothed data for average vapor compositions at 10, 20 . . . . 80 mole % calculate Y_{off} and Y_{on}. For example if $(Y_{off}+Y_{on})/2 = 10.0$ mole % and the corresponding $Y_{off}-Y_{on} = 15$ mole %, then $Y_{on} = 2.5$ mole % and $Y_{off} = 17.5$ mole %. Since the column is operated at total reflux, $X_{off} = Y_{on} = 2.5$ mole % and $Yx_{off}^* = 17.0$ mole % from the equilibrium diagram for methanol-water.

(b) In order to find m_G and m_L for each composition, the corresponding values of x_i and y_i must be found as described above. Assumed values at the reference concentration, for example $N_L/N_{G\,60} = 3.0$, are taken, and N_L/N_G for the other compositions are calculated with the A and B factors defined basically in equations but modified as discussed in the following paragraphs.

The A.I.Ch.E. group report the effect of physical properties on N_G of

$$N_G = N_{G\,60} \frac{(D_v^{0.5} T\, \rho_G^{0.13} t_G)}{(D_v^{0.5} T\, \rho_G^{0.13} t_G)_{60}} \quad (12)$$

From measurements of ϕ reported previously, it appears that t_G is not a function of composition. Furthermore the effect of density is seen to be small, and the effect of viscosity (although not yet reported by the A.I.Ch.E. group) would be expected to vary inversely with density. Therefore for the system methanol-water

$$B = \left(\frac{T_{60}}{T}\right)^{1.75} \quad (13)$$

The A factor is calculated from Equation (3). The exact proportionality between k_L and D_v is not known for a foaming tray but here has been taken to be proportional to $N_{SC}^{-0.5}$, primarily following the work of Danckwerts (*10*). The inclusion of ρ_L and μ_L in the Schmidt number is an assumption. It would be expected that a_L would be independent of composition, and in this work ϵ was found to be similarly independent. The volumetric flow rate is not constant but is inversely proportional to ρ_{ML}. From these arguments A is defined for the system methanol-water as

$$A = \left(\frac{\mu_L D_L}{\rho_{ML}}\right)_{60}^{0.5} \left(\frac{\rho_{ML}}{D_L \mu_L}\right)^{0.5} \frac{\rho_{ML\,60}}{\rho_{ML}} \quad (14)$$

The A and B correction factors are shown in Figure 7.

With the smoothed data (for example Figure 6) used, the calculations were made for the average tray compositions, at increments of 10% from 10 to 80 mole % methanol. Compositions below and above these values were not used because the concentration differences are so small and/or the slope of the equilibrium curve is so large that there is an undesirable magnification of errors.

(c) By means of the over-all concentration change and the smoothed data E_{MV}, E_{ML}, and m_T are calculated. Then N_{OG}', $1/N_{OG}'B$, N_{OL}', and $1/N_{OL}'A$ are calculated for each of the three special cases for the gas-phase driving force and for the two special cases for the liquid-phase driving force. Figures 8 and 9 show the plots of these curves and the extrapolations to $1/N_{G\,60}$ and $1/N_{L\,60}$ for these operating conditions. It should be noted that the $N_{L\,60}/N_{G\,60}$ calculated from these extrapolations must check the original assumption.

Now with $N_{G\,60}$ and $N_{L\,60}$ known, N_G and N_L at other compositions may be calculated by the use of B or A factors. In turn the point efficiency may be calculated by combining these individual-phase transfer units by means of Equations (1) and (6). The values of E_{MV} and the calculated E_{OV} are shown in Figure 10 together with curves for other velocities for this tray geometry. This figure demonstrates the effect of the liquid gradients on the tray on E_{MV}.

Figure 10 indicates several interesting effects of changing methanol concentration upon E_{OV} and E_{MV}. E_{OV} is seen to decrease at lower concentration. This is believed to be caused by the corresponding increase in m_G resulting in the liquid-phase resistance becoming appreciable.

At the same time E_{MV} increases with decreasing methanol concentration. This is believed to be the result of increasing the liquid concentration change across a given tray at these lower concentrations. Thus it could be expected that this increased concentration change would increase the concentration gradient, decreasing the effects of mixing and therefore increasing E_{MV}. Presumably if reliable data were available at concentrations lower than those shown on Figure 10, it could be shown that E_{MV} approaches E_{OG} as the methanol concentration approaches zero and the liquid concentration change across the plate approaches zero.

It is obvious that the liquid concentration change across the tray is governed by the shape of the equilibrium curve for total reflux operations. Thus the plot of E_{MV} against concentration for any system under total reflux condition is dependent upon the shape of the equilibrium curve.

It is obvious from Figure 9 that the extrapolation to $1/N_{LS}$ is questionable because of the difficulty of extrapolation and because the actual degree of mixing is unknown. However $1/N_L$ is small and is significant only at high values of m_G. In this work the extrapolation of Equation (8) was used for N_L because, for the system used, increasing m_L in general corresponds to increasing values of $Y_{on} - Y_{off}$, and therefore the tray performance is approaching that for no liquid mixing. At the same time the extrapolation of $1/N_L$ is not considered sufficiently accurate to attempt a more fundamental analysis of N_L such as will be considered for N_G.

THE EFFECT OF GAS VELOCITY ON N_G

When one follows the mass transfer theory outlined above, the N_G values can be treated further. From the definition of N_G it is apparent that N_G is not a function of liquid rate, provided that β does not vary with liquid rate. The effect of liquid rate on Z_v has been shown to be very small (*1*), and therefore little effect on β would be expected.

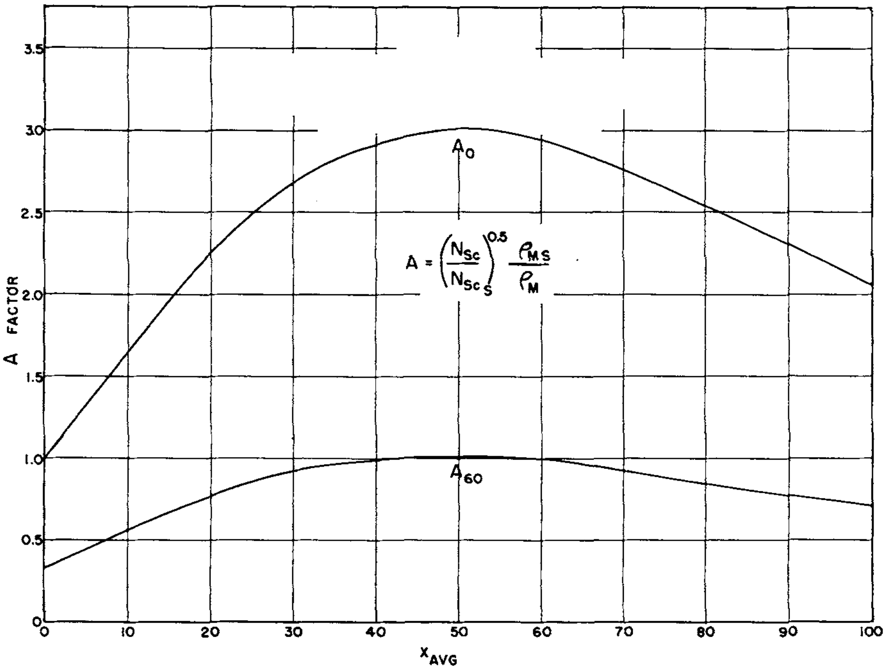

Fig. 7a. *A* factor vs. liquid composition for methanol-water.

Theoretically if the assumptions outlined in the introduction are applicable, a plot of N_G against β/U should yield a straight line passing through the origin with slope $k_G'a_G$, if $k_G'a_G$ is not a function of gas velocity. Such a plot is presented in Figure 11 for a bulk composition of vapor and liquid of 60 mole % methanol. Also included on the plot are the data of Price (*30*) for 3- and 4-in. weirs with the same apparatus, system, and calculation methods.

Some care must be taken in drawing conclusions from such a plot since it is difficult to measure β/U with accuracy. It has been shown that a given weir height ϕ is practically constant with gas velocity, and therefore the time of contact is proportional to Z_v. The foam height does increase with gas velocity, and therefore the time of contact does not decrease proportionately with increasing gas rate.

A DISCUSSION OF TIME OF CONTACT

It is evident that the effect on N_G of vapor velocity depends to a large ex-

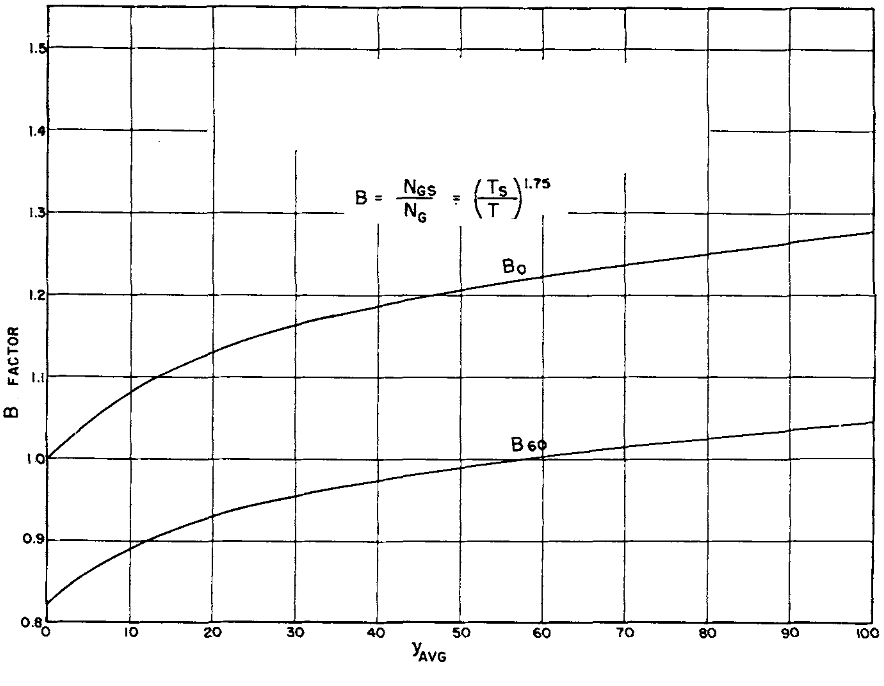

Fig. 7b. *B* factor vs. vapor composition for methanol-water.

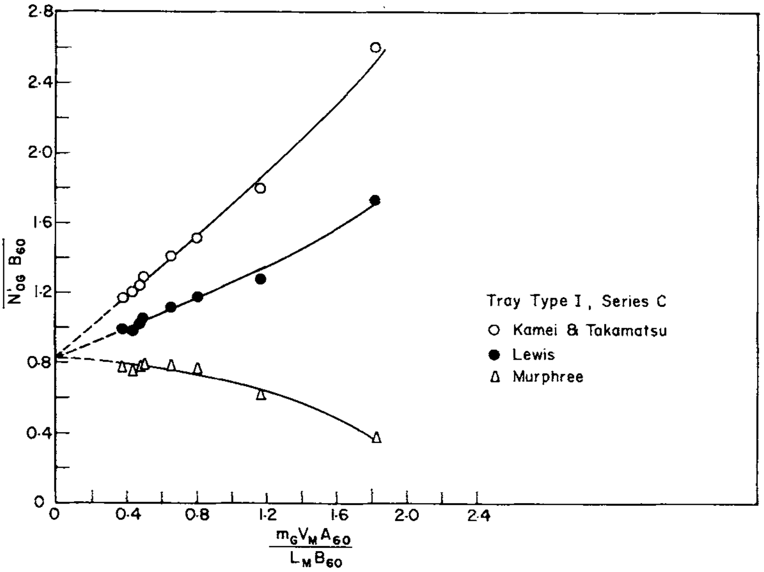

Fig. 8. Extrapolation to $(1/N_G)_{60}$, reference concentration 60 mole % methanol.

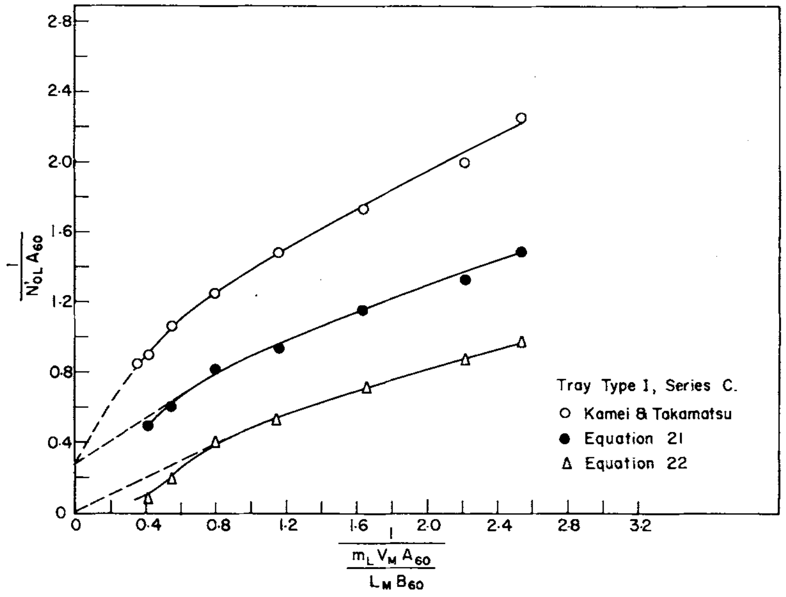

Fig. 9. Extrapolation to $(1/N_L)_{60}$, reference concentration 60 mole % methanol.

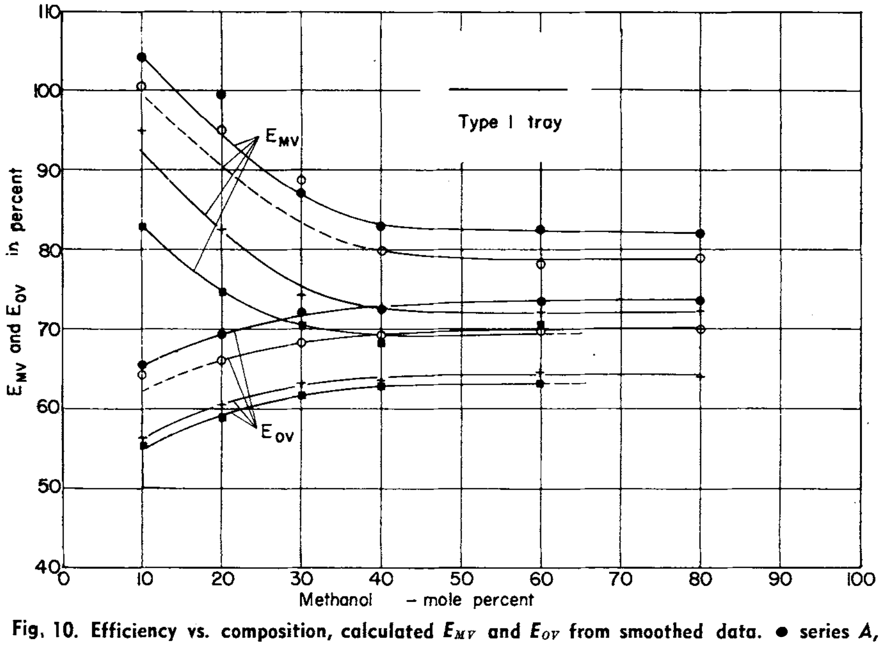

Fig. 10. Efficiency vs. composition, calculated E_{MV} and E_{OV} from smoothed data. ● series *A*, o series *B*, + series *C*, ■ series *D*.

tent on the definition of the time of contact for which several relationships have been proposed. Geddes (*14*) suggested that bubbles rose through the foam at the terminal velocity of a freely rising bubble, and therefore the time of contact would be independent of vapor velocity. Several investigators (*7*, *12*) have supported this idea, but the data were taken on bubble-plate trays in which there might be relatively little interference between bubbles from adjacent slots.

Stone (*33*) has proposed that the bubbles actually push each other through the foam at a velocity proportional to the superficial gas velocity, and therefore the time of contact is proportional to Z_v/U. Using both bubble-cap and sieve trays operating at high gas velocity, he found support for this relationship. Anderson's data (*2*) on sieve trays also supported Stone's mechanism for weir heights of 2 in. In this paper the contact time proposed by Gerster is used:

$$t_G = \frac{\beta}{U} = (1-\phi)\frac{Z_v}{U} \qquad (15)$$

Except for the obvious difficulty of measuring ϕ at low seals, it provides an accurate definition of t_G over the range of interest.

t_G having been defined in this manner, the data reported here shows that $k_G' \, a_G$ is a function of tray geometry but not of gas velocity. This is different from the results of the A.I.Ch.E. group (*1*), which used bubble trays where $k_G' \, a_G$ is a function of gas velocity up to a limiting value. It is suggested here that the latter effect is due to increasing a_G; with increasing gas velocity the bubbles tend to break up until they reach a minimum average size at the limiting velocity. As a part of this study Price (*30*) has found that ϕ varies with weir height, and therefore the time of contact is not proportional to Z_v/U if a number of weir heights are to be considered. However on Figure 11 this effect does not appear when time of contact is defined as β/U; for each tray geometry a straight line through the origin is seen to represent the data adequately.

THE EFFECT OF TRAY GEOMETRY

The effect of tray geometry is reflected by the slopes of the lines on Figure 11. Type I trays have larger $k_G' \, a_G$ values than the other two types, but types II and III have almost indistinguishable slopes.

The experimental evidence that is available to support any explanation of mass transfer within the foam is restricted to foam density and foam heights, both of which are difficult to measure exactly. These data do not

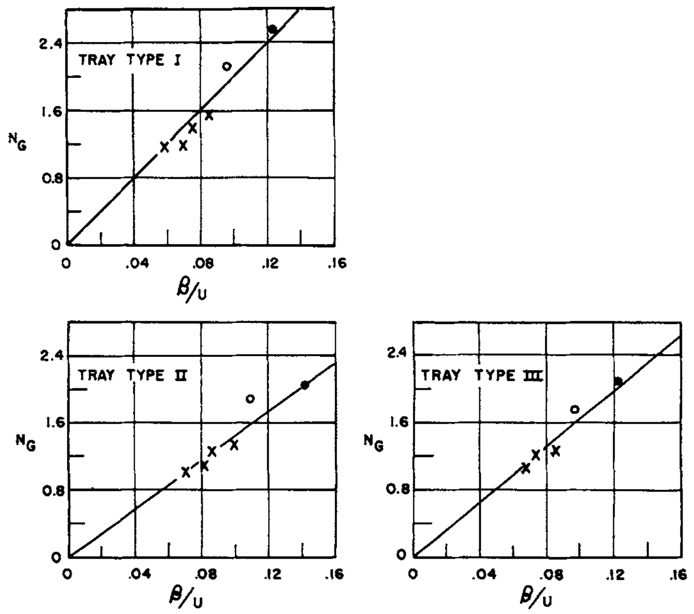

Fig. 11. Number of gas phase transfer units vs. time of contact. ● 4-in. weirs, o 3-in. weirs, x 2-in. weirs.

indicate changes in interfacial area, since the number and size of bubbles could be changing without affecting foam density. The bubble shape also changes, but the assumption of essentially spherical bubbles is generally made to affect some simplification; photographic studies have indicated that this is reasonable (*11*).

The performance of type I and type III trays which have the same free area can be compared to indicate the effect of hole size. The foam density is seen to be constant for both trays, and $k_G' \ a_G$ decreases with increasing hole size. This decrease is believed to be due primarily to a decrease in a_G with increasing hole size rather than a decrease in the turbulence and therefore a decrease in k_G'.

With type II tray the free area was increased. It is possible that the time of contact was increased while $k_G' \ a_G$ was decreased.

END EFFECTS

Work with single bubbles (*9*, *18*, *31*) and the study of Anderson (*2*) have shown that k_G is probably higher at formation than during bubble rise. Anderson has noted that this effect is not apparent at higher seals, since the bubble a_G at formation is smaller than in the foam, probably owing to bubble breakup. The data on Figure 11 and the work of the A.I.Ch.E. group (*1*) support the assumption that end effects are small.

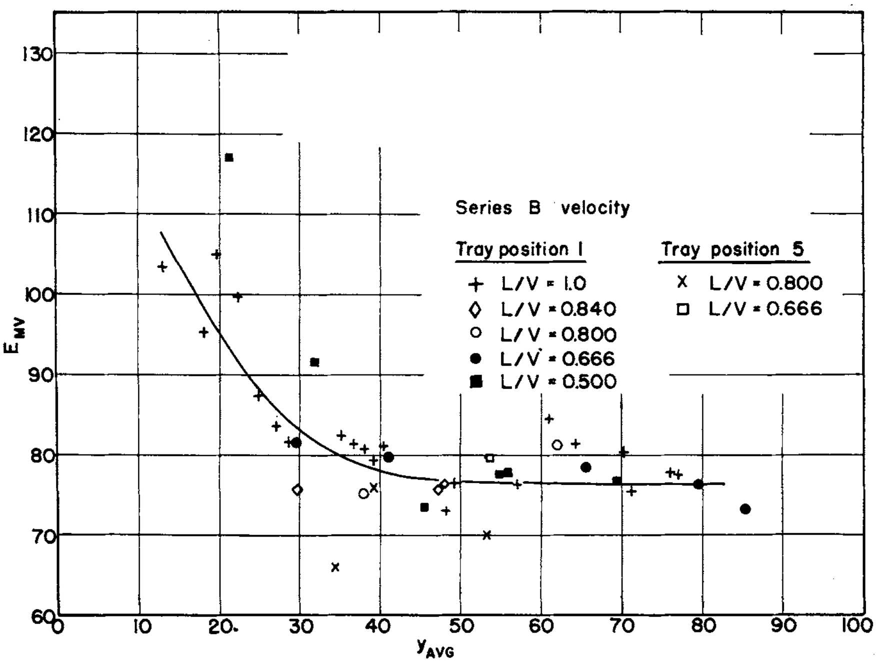

Fig. 12. E_{MV} vs. vapor composition for various L_M/V_M.

EFFECT OF REFLUX RATIO

A few runs were made at reflux conditions other than total reflux and are reported on Figure 12. For a given tray geometry and change in liquid concentration E_{MV} was found to be practically independent of reflux ratio. It was found empirically that less scatter of the data was obtained by using Y_{avg} than using X_{avg} in plotting the points on this graph, Y_{avg} and X_{avg} no longer being equal as for the total reflux runs. It is recommended that the point efficiency be calculated as for a total reflux run and that the appropriate mixing Equation (*24*, *26*) be used to yield E_{MV} for a tray under any operating condition.

CONCLUDING REMARKS

This paper has provided new efficiency data for the operation of sieve plates in an 8-in.–diameter column on the system methanol-water. It is believed that the values of N_G are the first reported in the literature for a tray in distillation operation.

ACKNOWLEDGMENT

The authors would like to acknowledge the work of Price (*30*), who checked some of the original data and carried out the investigation of 3-and 4-in. weir heights.

NOTATION

A = correction factor for N_L to standard set of conditions (60 mole % methanol)
a = interfacial area per unit volume of foam, sq. ft./cu. ft.
a_G = interfacial area per unit volume of vapor, sq. ft./cu. ft.
a_L = interfacial area per unit volume of liquid, sq. ft./cu. ft.
B = correction factor for N_G to standard set of conditions (60 mole % methanol)
b = constant
c = concentration, lb. moles/cu. ft.
D_L = diffusivity in the liquid, sq. cm./sec.
D_V = diffusivity in the vapor, sq. cm./sec.
E_{MV} = Murphree plate efficiency based on vapor driving force
E_{ML} = plate efficiency based on liquid driving force
E_{OV} = point or local efficiency based on vapor driving force

K_G = over-all mass transfer coefficient based on vapor driving force, lb.moles/(sq.ft.) (sec.) (atm.)
K_L = over-all mass transfer coefficient based on liquid driving force, lb. moles/(sec.) (sq. ft.) (lb. mole/cu.ft.)
k_G = individual gas-phase mass transfer coefficient, lb.moles/(sec.) (sq.ft.) (atm.)
k_G' = $k_G\ P/\rho_{MG}$ or $k_G RT$, ft./sec.
k_L = individual liquid-phase mass transfer coefficients, lb. moles/(sec.) (sq. ft.) (lb. mole/cu. ft.)
L_M = liquid molar flow, moles/sec.
L_{OY} = length of over-all gas phase transfer exit, ft.
M = molecular weight
m = slope of equilibrium line
m_G = slope of equilibrium line with calculations based on vapor driving force defined by Equation (15)
m_L = slope of equilibrium line with calculations based on liquid driving force defined by Equation (15)
m_T = slope of equilibrium line defined by Lewis to relate point and plate efficiencies on a tray
m_T' = slope of equilibrium line defined by Kamei and Takamatsu to relate N_{OG} and E_{MV} on a plate
N_G = number of individual vapor-phase mass transfer units = $\dfrac{k_G PaZ_V}{\rho_{MG}U} = k_G RTa_G\dfrac{\beta}{U} = k_G'at_G$
N_L = number of individual liquid-phase mass transfer units = $k_L a\,\epsilon/q = k_L a_L t_L$
N_{OG} = number of over-all gas-phase mass transfer units = $K_G PaZ_V/\rho_{MG}U$
N_{OL} = number of over-all liquid-phase mass transfer units = $K_L\rho_{ML}aZ_V/L_M$
N_{SC} = Schmidt number $\mu_L/D_L\rho_L$
P = pressure, atm.
q = total volumetric flow of liquid across tray, cu. ft./sec.
T = absolute temperature
t_e = time of contact in general solution of unsteady state diffusion equation applied to liquid at interface of a bubble, sec.
t_G = time of contact of vapor with liquid on tray, sec.
t_L = time of contact of liquid with vapor on tray
U = linear velocity of vapor within the column based on the bubbling area, ft./sec.
V_M = molar vapor flow, moles/sec.
X = average mole fraction more volatile in the liquid entering or leaving the tray
x = mole fraction more volatile in the liquid at a particular point on the tray
Y = average mole fraction more volatile in vapor entering or leaving the tray
y = mole fraction more volatile in vapor at a particular point on the tray
Z_V = height of foam, ft.

Greek Letters

β = volumetric holdup of vapor in foam, cu. ft./sq. ft. of tray bubbling area
ϵ = total volumetric holdup of liquid on tray, cu. ft.
$\Delta P_f'$ = pressure drop through foam on tray corresponding to clear liquid head, cm. of liquid on tray
ΔP_f = total pressure drop through the tray cm. of water
ΔP_R = difference of pressure drop through foam and clear liquid head on tray
ρ = density, lb./cu. ft.
ρ_M = molar density, lb. moles/cu. ft.
ϕ = volume fraction of clear liquid on tray, cu. ft. of liquid/cu. ft. of foam
ϕ' = $\Delta P_f'/Z_V$
μ = viscosity of liquid

Subscripts

G = gas or vapor phase
i = concentrations at the interface
L = liquid phase
M = quantities on molar basis
on, off = compositions entering or leaving tray respectively
S = standard or reference concentration to which N_G and N_L at other concentrations and physical properties are corrected
X_{on}, X_{off}, Y_{on}, Y_{off} = material to which the equilibrium refers; $Y^*_{X_{on}}$ = the mole fraction more volatile in a hypothetical vapor in equilibrium with the liquid entering the tray, used with superscript *

Superscript

* = phrase *mole fraction more volatile in a hypothetical liquid in equilibrium with;* material with which equilibrium occurs is denoted by the subscript

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